# Dynamics Final Project: A Study of Passive Dynamic Walking using a Point Mass Model

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**Abstract**

Passive Dynamic Walkers are unactuated bipeds that use gravity to walk down a shallow slope. The structures that currently exist throughout the world are often used to imitate the dynamic motion of the human gait. In this project, we modeled a simple point-mass, bipedal walker in MATLAB using equations of motion outlined by Garcia et al in their paper "The Simplest Walking Model: Stability, Complexity, and Scaling". Our simulation results indicate that stable gait-cycles exist for ramp angles of approximately 0.009 rad to 0.013 rad. Outside of this range, the passive dynamic walker either fails immediately or is unable to maintain forward motion for more than a few steps. Within the stable range, our simulated walker is capable of walking for more than 16000 steps without failing.

## 1 Background

Passive Dynamic Walkers are a fascination in the science and engineering worlds, particularly in the realm of robotics. They are the most efficient walking motion as they do not require an actuation assembly or powered input. They instead use gravity as their sole energy input mechanism. They are also some of the most accurate models for bipedal locomotion as they mimic the dynamic 'falling forward' motion that characterizes the human gait. These systems however are extremely delicate when creating a mechanical model so they pose an interesting challenge to researchers and engineers throughout the world.

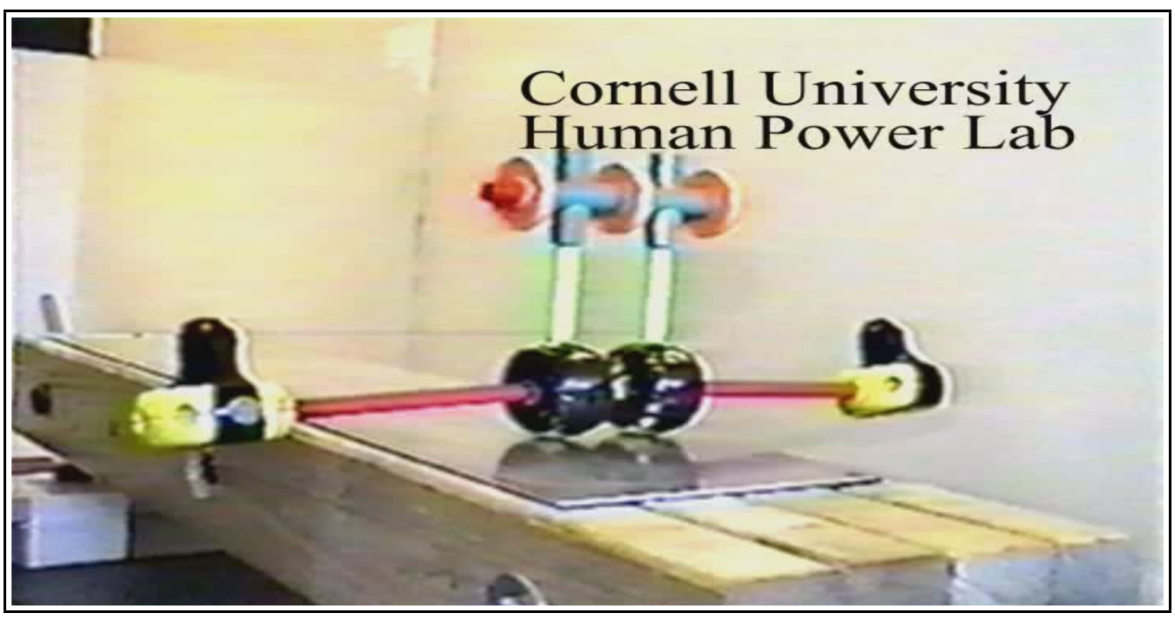


Figure 1: The TinkerToy Passive Dynamic Walker created by Garcia, Mombauer, Coleman, and Ruina

In the paper, "An Uncontrolled Toy That Can Walk but Cannot Stand Still" by Coleman and Ruina, a 3D model is created to approximate the TinkerToy passive dynamic walker using each body's moment of inertia and Euler Angles.

This project drew heavily from the paper "The Simplest Walking Model: Stability, Complexity, and Scaling" by Garcia, Chatterjee, Ruina, and Coleman. This paper presents a 2D, two-link passive dynamic walker that can walk down a shallow slope solely under the influence of gravity and can therefore also be used to approximate the behavior of the TinkerToy walker. This passive dynamic walker is characterized by a point-mass at the hip and negligible point-masses at the feet, allowing Garcia et al. to approximate the walker's motion using equations derived from angular momentum balances about each point mass. "The Simplest Walking Model" also presents analytical solutions for certain cases of their simplified model.

Ted McGeer's paper "Passive Dynamic Walking" was also used as a reference for this project. This paper introduces and explores both kneed and straight-legged passive dynamic walkers, comparing simulated behaviors of both. It also discusses experimental data from passive dynamic walkers and compares it to the simulated behavior.

## 2 Learning Objectives

Initially, we had two primary learning objectives for this project: to better understand how physical parameters (notably ramp angle) affect passive dynamic walking behavior, and to determine how point-mass and rigid-body models differ in accuracy and stability when compared to empirical data. We were also interested in seeing whether passive dynamic walkers could walk down a flight of stairs and if so, what the limiting parameters of the staircase and walker were.

As we learned that passive dynamic walker behavior is entirely dependent on the initial conditions and physical parameters of the system, we decided to focus on our first learning objective. We abandoned the rigid-body model entirely as we found it easier to see the effects of initial parameters with the simplified point-mass model.

## 3 System Model

The model that we used models the legs and feet of the walker as two point-mass pendulums, connected at the hip. It assumes that the weight of the legs and feet can be approximated as identical point masses on the feet. It also assumes that the mass of the hip is larger than the masses of the feet. It assumes that the walker is walking down a constant grade slope. It also assumes that for this motion to be possible, the walking foot must temporarily go through the floor as it passes the standing foot.

This model measures the angle between the stance foot and the normal to the ramp ($\theta$) and the angle between the stance leg and the swinging leg ($\phi$). $M$ denotes the mass of the hip and $m$ denotes the mass of each foot. $\psi$ is the angle of the ramp. Each leg has a length $l$. Note that g is the acceleration due to gravity in the downwards direction.

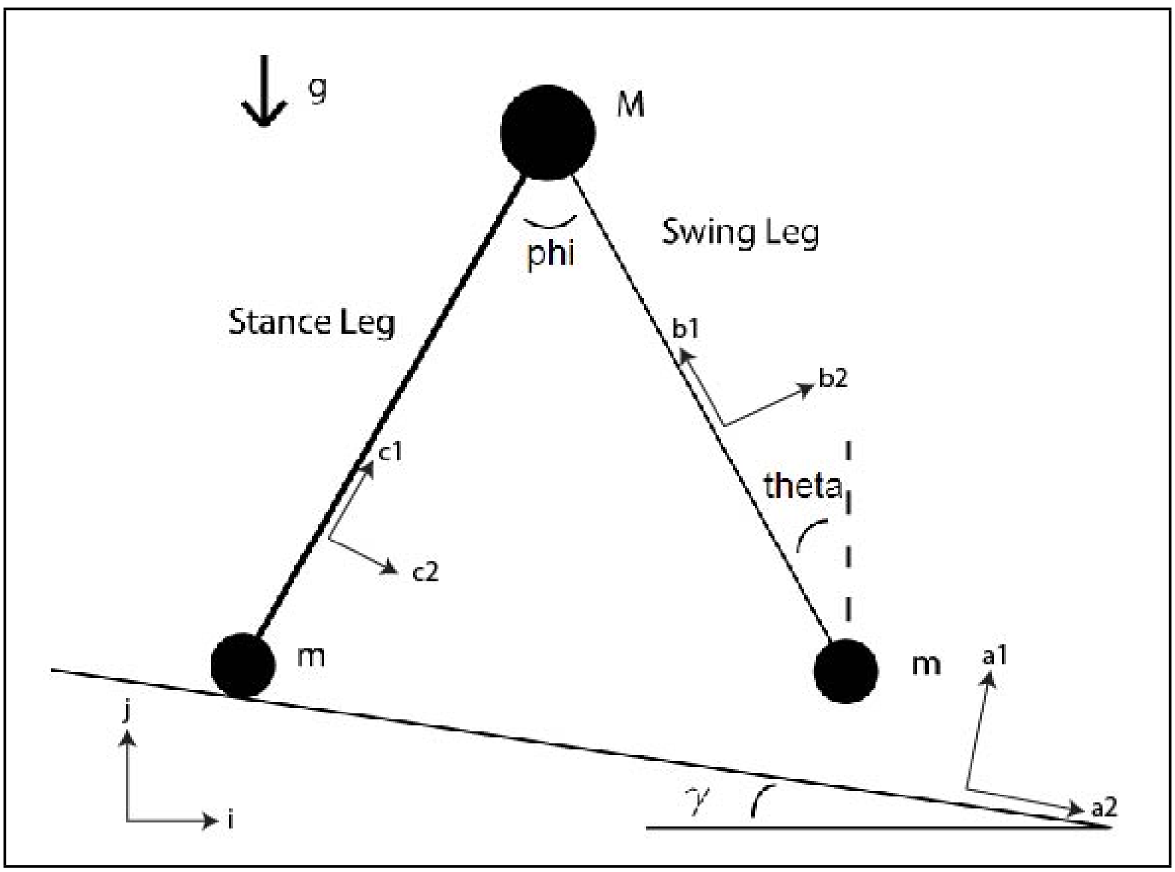


Figure 2: The diagram explaining the parameters of "The Simplest Walking Model"

Note that in Figure 2, various reference frames are included. The Cartesian reference frame (i-j) has its origin about the beginning of the bottom of the ramp. The a-frame has the same origin as the Cartesian frame (but is drawn elsewhere for a clearer image). a1 is normal to the ramp while a2 is parallel to the ramp. The b-frame has its origin about the hip mass (M) where b1 is parallel to the swing leg (note that the swing leg switches as the toy walks). The c-frame has its origin about the hip mass (M) and c1 is parallel to stance leg.

The following transformations were necessary to to bring everything into the Cartesian reference frame.

The $\hat{c}$-frame to $\hat{b}$-frame:

$$\begin{bmatrix} \hat{b1} \\ \hat{b2} \end{bmatrix} = \begin{bmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} \hat{c1} \\ \hat{c2} \end{bmatrix}$$

The $\hat{b}$-frame to $\hat{a}$-frame

$$\begin{bmatrix} \hat{a1} \\ \hat{a2} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} \hat{b1} \\ \hat{b2} \end{bmatrix}$$

The $\hat{a}$-frame to Cartesian:

$$\begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} = \begin{bmatrix} cos\gamma & sin\gamma \\ -sin\gamma & cos\gamma \end{bmatrix} \begin{bmatrix} \hat{a1} \\ \hat{a2} \end{bmatrix}$$

The following parameters were examined throughout the simulation. Constant parameters are given below:

$M$ : 1.9 (the mass of the hip in kg)
$m$ : 0.05 (the mass of the foot in kg)
$l$ : 1.568 (the length of the leg in m)
$\phi$: the angle of the standing leg with respect to the normal of the slope(in rad)
$\theta$: the angle of the walking leg with respect to the standing leg (in rad)
$\gamma$ : the angle of the ramp (in rad)
$g$ : 9.8 (acceleration due to gravity m/s$^2$)

As the walking leg is swinging, the angles $\theta$ and $\phi$ are governed by the following set of ordinary differential equations for a double pendulum:

The equations represent the balance of angular momentum about the standing foot with $\theta$ for the entire mechanism and about the hip with $\phi$ for the walking leg. These equations of motion hold for the entirety of the simulation. However, there must be a transition point where the conditions are changed such that the walking and standing legs are switched via conversation of angular momentum.

$$\begin{bmatrix} 1+2\beta(1-cos\phi) & -\beta(1-cos\phi) \\ \beta(1-cos\phi) & -\beta \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} -\beta sin\phi(\dot{\phi}^2 - 2\dot{\theta}\dot{\phi}) \\ \beta\dot{\theta}^2 sin\phi \end{bmatrix} + \begin{bmatrix} (\beta g/l)[sin(\theta-\phi-\gamma) - sin(\theta-\gamma)] - g/l sin(\theta-\gamma) \\ (\beta g/l) sin(\theta-\phi-\gamma) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

This system goes through a transition where the swing leg switches lands on the ground and switches with the stance leg.

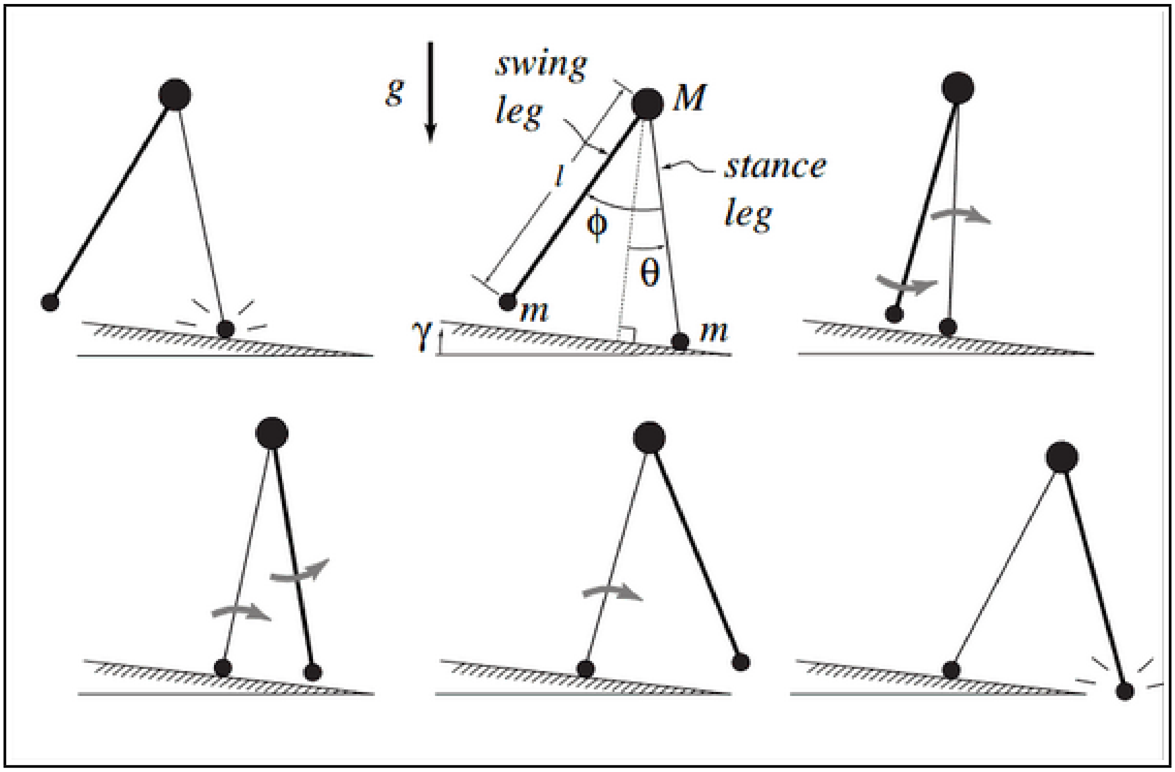


Figure 3: The swing leg (in bold) swings from the past the stance leg and lands on the slope. At the transition, the system goes through a transition to denote the change in stance leg.

# 4 Results and Analysis

The following simulations were conducted using the constant parameters given previously in "System Model". Our initial conditions were as follows:

$\theta_0 = 0.2$ (initial angle of stance leg to ramp normal in rad)
$\phi_0 = 0.4$ (initial angle between stance and swing legs in rad)
$\dot{\theta} =$ -0.5289 (initial rate-change of stance angle to ramp normal in rad/s)
$\dot{\phi} = 0$ (initial rate-change of angle between stance and swing legs in rad/s)

## 4.1 Comparison to Reference Literature

The model that we programmed turned out to be a very robust, fast model. We validated our model by inputting parameters provided in the paper by Garcia et al. and comparing the results of their model to ours. Our results were extremely consistent with those found in the "Simplest Walking Model" and"Passive Dynamic Walking" by McGreer.

Due to the assumptions made, the "Simplest Walking Model" normalized time for $(l/g)^2$. Time is not normalized in our model, as we do not follow the same simplifying assumptions/equations that Garcia et al do. Despite this fact, our comparison of the published model's characteristic heel scuff with our model's heel scuff demonstrated that our system was exhibiting the proper behavior.

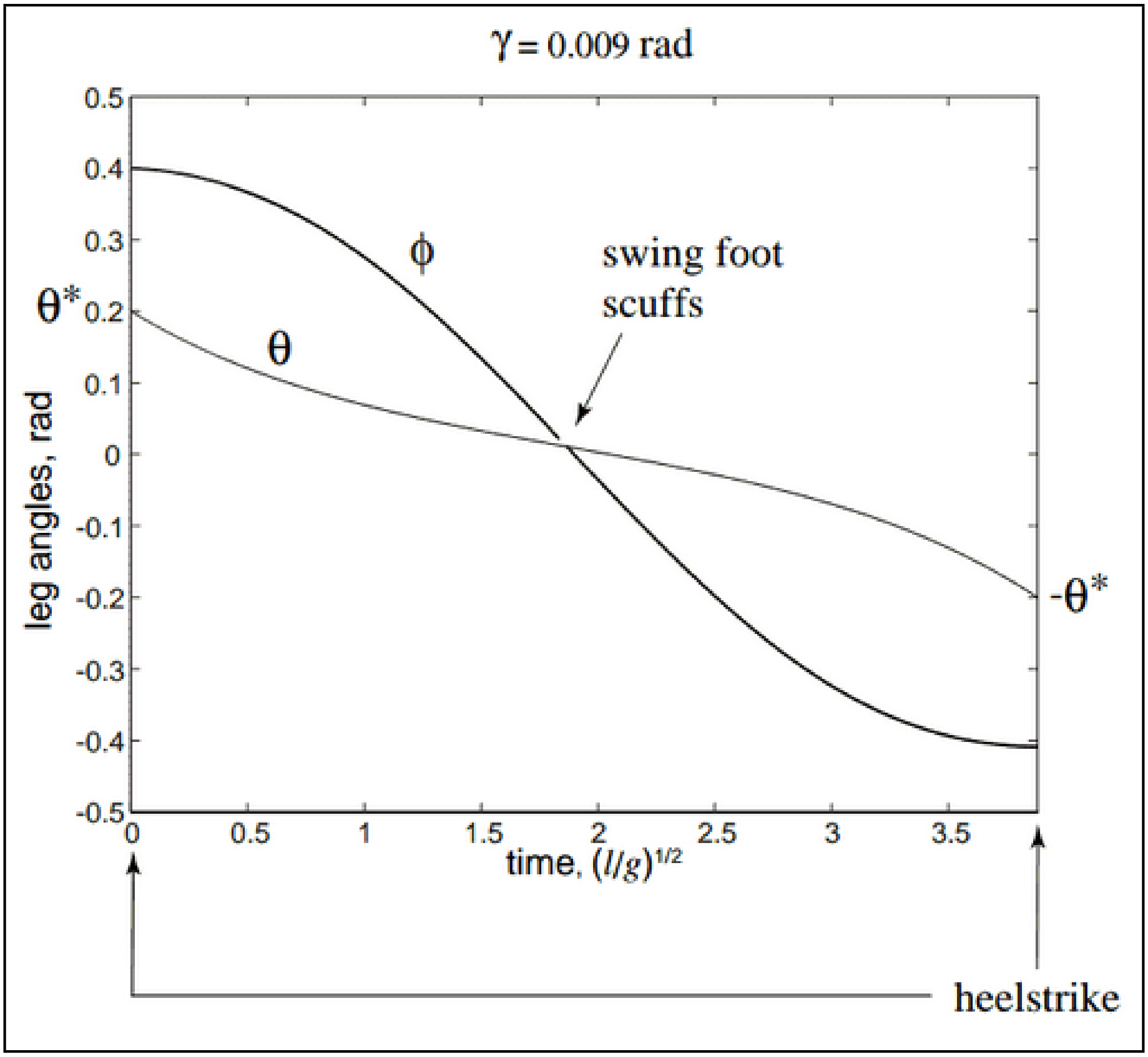


Figure 4: The heel scuff event as described by Garcia et al.

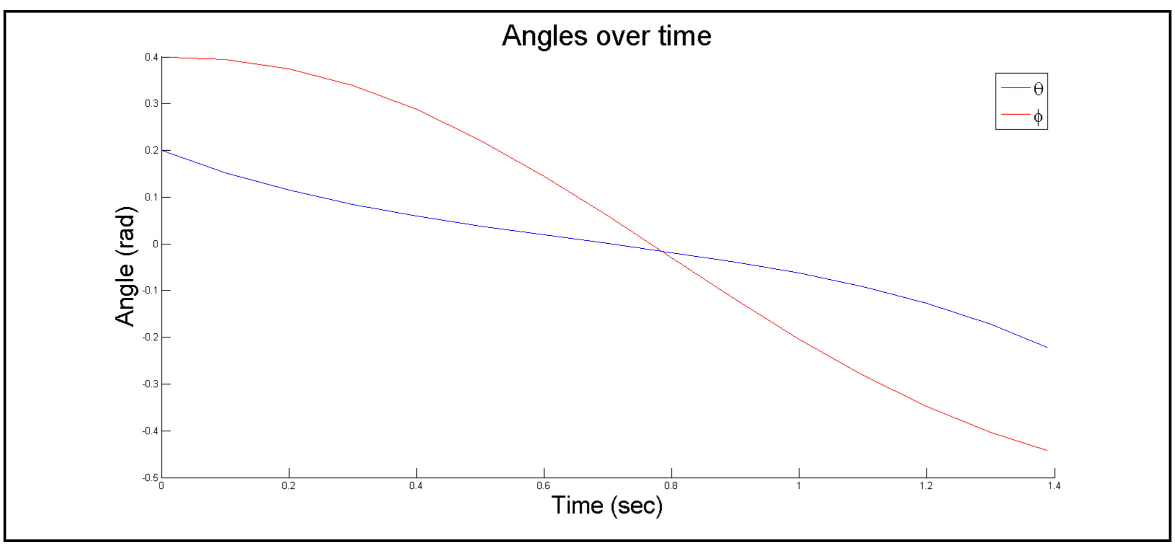


Figure 5: Our model's heel scuff event. Note that the characteristic shape and even similar values are present.

## 4.2 Effect of Initial Parameters on Stability

Through our model, we discovered that the initial parameters of the system had a great effect on the stability (i.e. its ability to create a reasonable cycle of motion for the walker) of the system. We first examined the effects of varying the ramp angle $\gamma$. Note that in Figure 6, $\gamma$ is so small that the walker is unable to maintain a plausible walking pattern because there is not enough potential energy for the system to walk down the slope. As a result, the model breaks down and returns bogus results. In Figure 7, the angular velocities reflect this issue as well. At this tiny ramp angle, the walker is able to complete only one step. Ultimately, only ramp angles between approximately 0.009 radians and 0.013 radians resulted in successful passive dynamic walking.

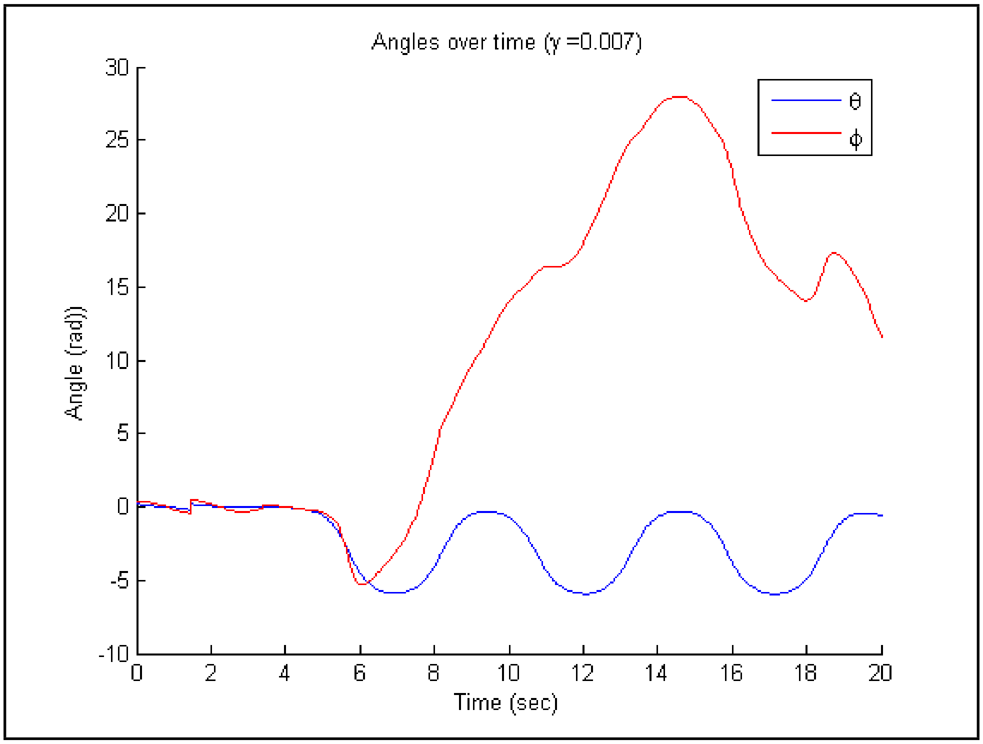


Figure 6: Simulation results for a ramp angle of 0.007 radians. The walker achieves one step in the first four seconds, but is otherwise unstable.

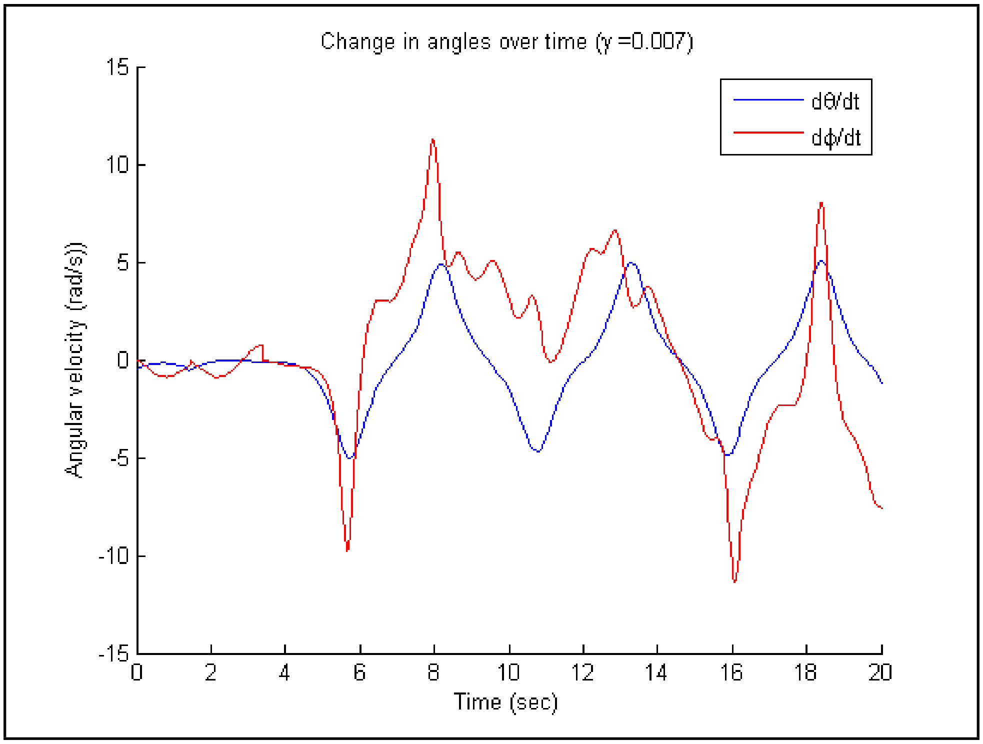


Figure 7: Simulation results for a ramp angle of 0.007 radians, showing the change in $\phi$ and $\theta$. Angular velocities become irregular after four seconds.

In the case of $\gamma$=0.009 and 0.013, it is clear that we are in the model's optimal zone. The system is cyclic within angular displacements and velocities that are expected. As an interesting side note, the model seems to indicate an issue period of irregularity but eventually evens itself out after about 8 steps. This may have been due to ODE45's choices of time step or perhaps more compellingly, the system may reflect the walker's initial instability due to imperfect initial conditions.

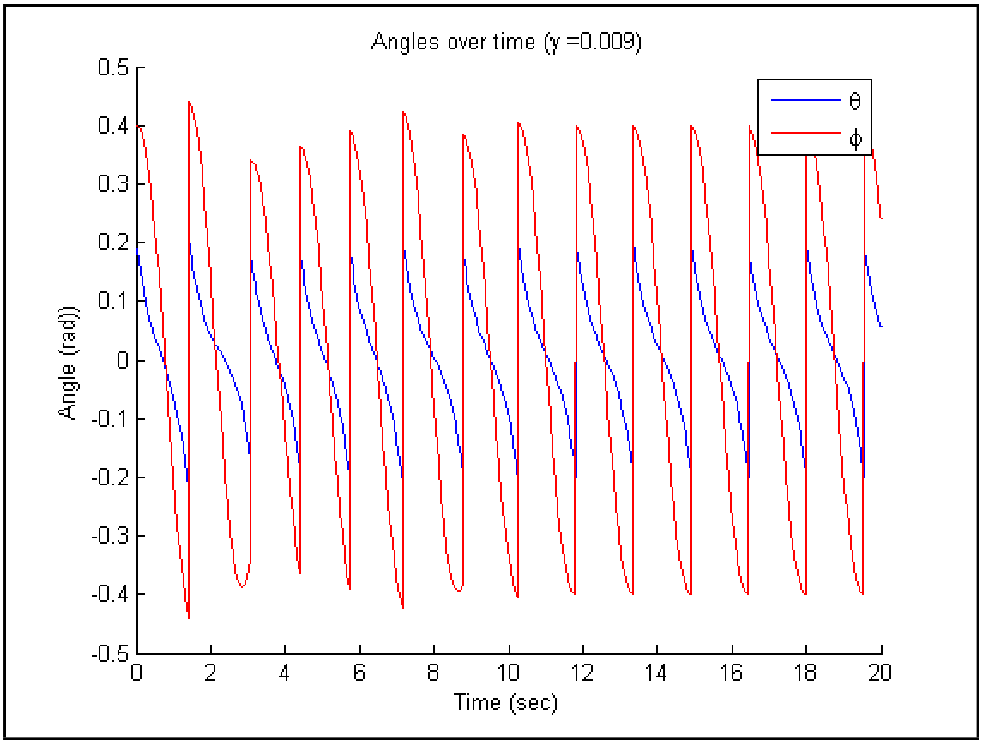


Figure 8: Simulation results for a ramp angle of 0.009 radians, showing $\phi$ and $\theta$ over seven full steps. Heel strike occurs at a consistent frequency, and both $\phi$ and $\theta$ settle into a repeated amplitude behavior after about ten seconds.

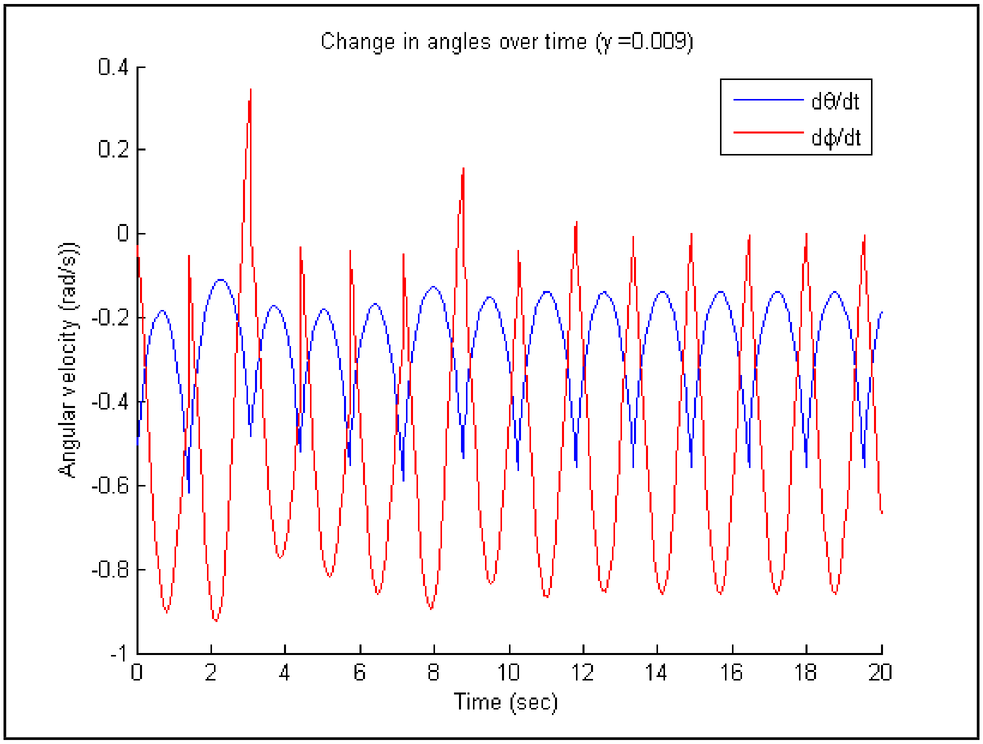


Figure 9: Simulation results for a ramp angle of 0.009 radians, showing the change in $\phi$ and $\theta$ over seven full steps. At heel strike, d$\theta$/dt is at a local minimum and d$\phi$/dt is at a local maximum. Amplitudes of d$\theta$/dt and d$\phi$/dt are regular after approximately ten seconds.

For angles greater than $\gamma$=0.013, the system again become unstable. However, we would like to note that the system is actually able to maintain some sort of cyclic response before becoming completely unreasonable. This may be because there is enough energy for the walker to perform, but due to the other parameters, the model breaks.

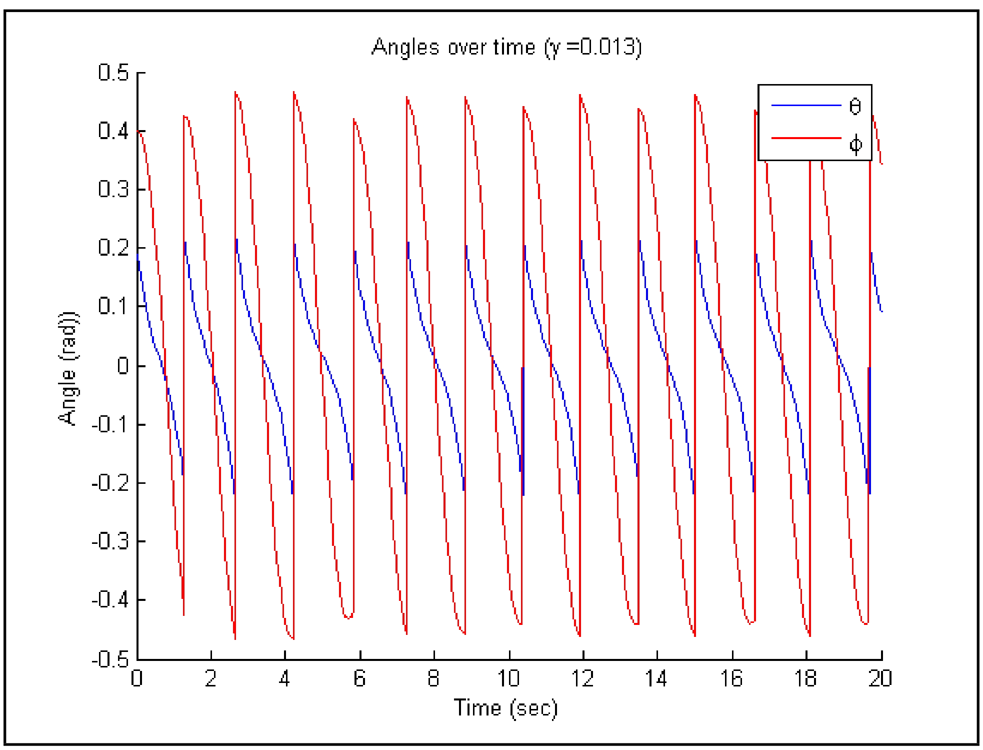


Figure 10: Simulation results for a ramp angle of 0.013 radians, showing $\phi$ and $\theta$ over seven full steps. Heel strike occurs at a consistent frequency, and the amplitudes of $\phi$ and $\theta$ follow a regular oscillating pattern with a period of two steps.

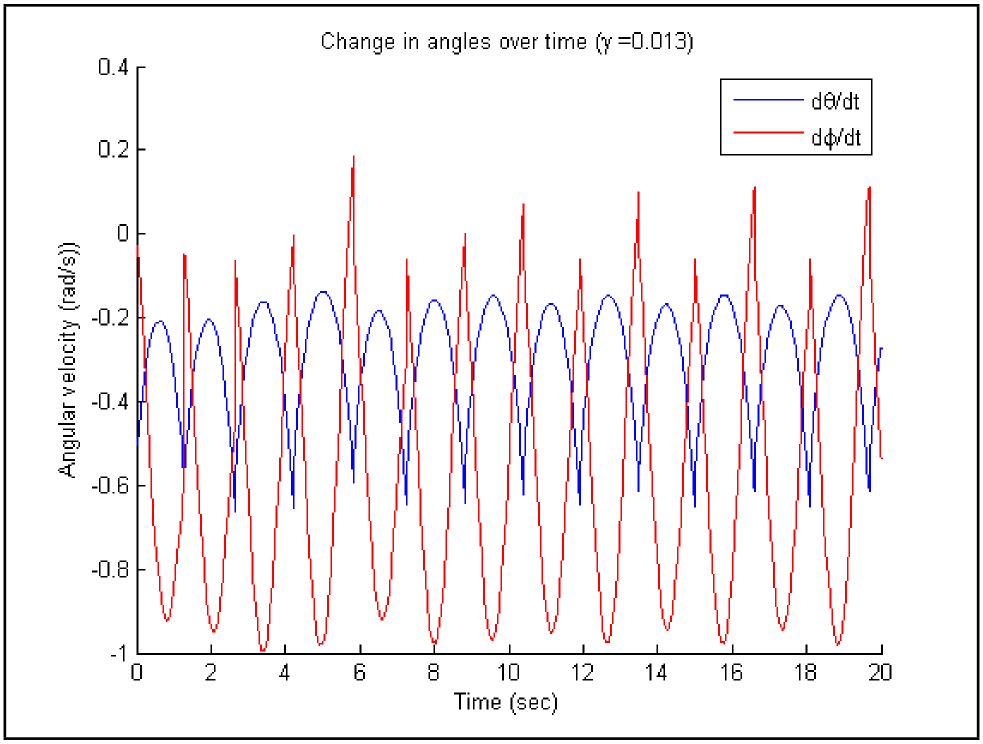


Figure 11: Simulation results for a ramp angle of 0.013 radians, showing $\phi$ and $\theta$ over seven full steps. At heel strike, d$\theta$/dt is at a local minimum and d$\phi$/dt is at a local maximum. Amplitudes of d$\theta$/dt and d$\phi$/dt follow the same oscillating pattern with a period of two steps.

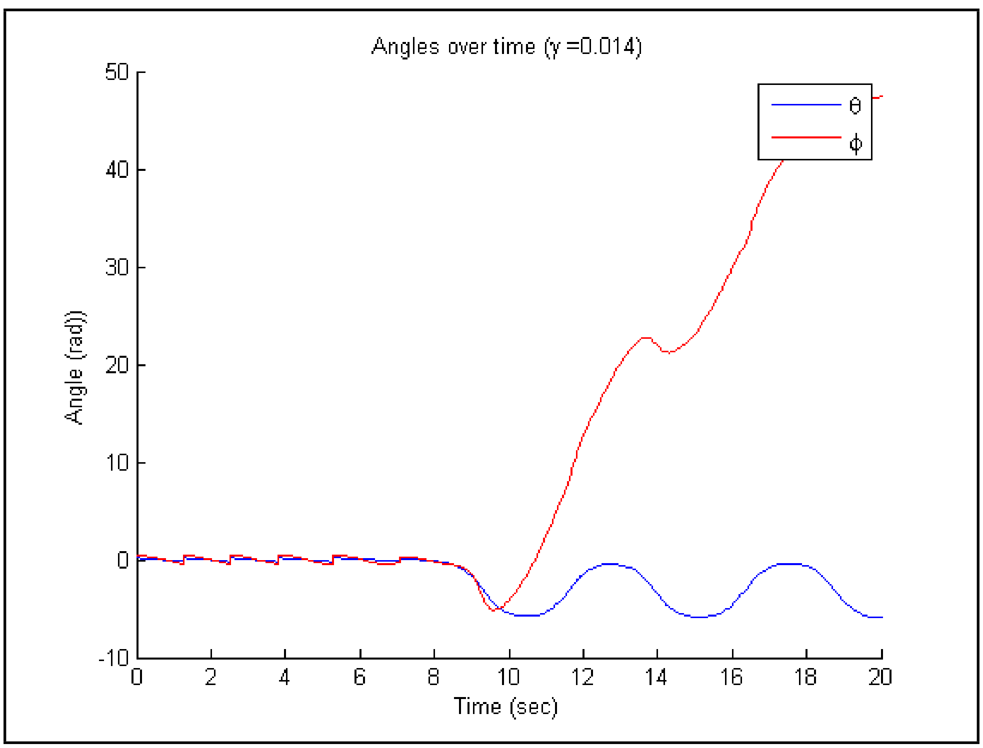


Figure 12: Simulation results for a ramp angle of 0.014 radians. The walker achieves three steps showing before failing.

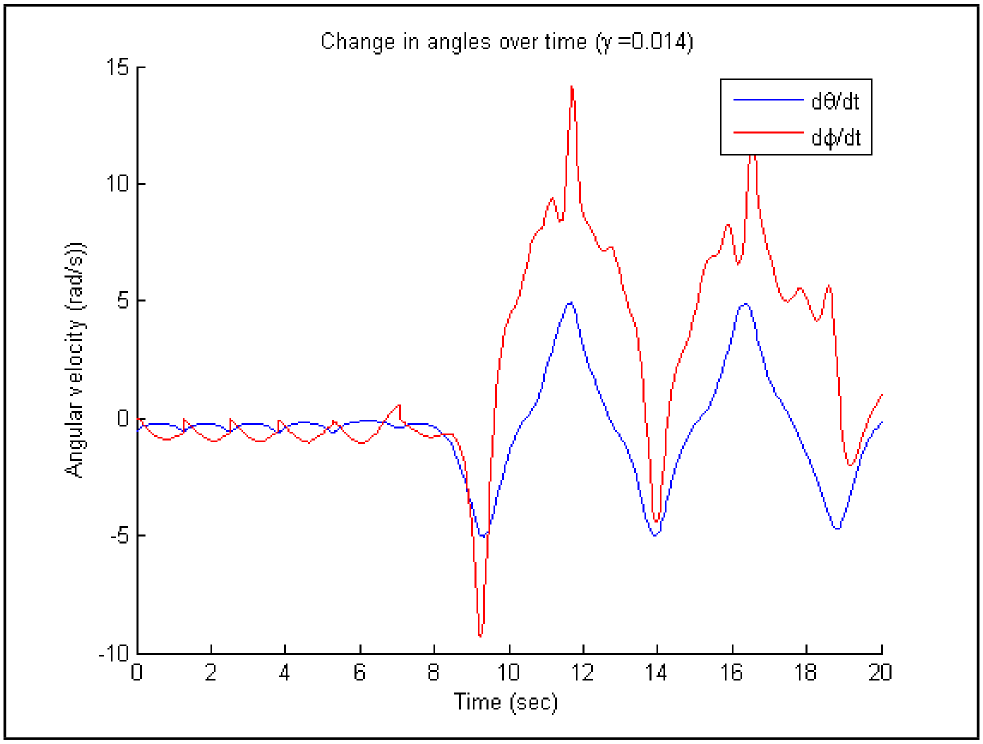


Figure 13: Simulation results for a ramp angle of 0.014 radians, showing the change in $\phi$ and $\theta$. d$\theta$/dt and d$\phi$/dt show consistent oscillating behavior until walker failure at the end of three steps.

## 4.3 Numerical inaccuracy

In our program, we noticed that the system was not always regular. In the plot below, we demonstrate the slight drift of the variables throughout the simulation. We believe this may be because ODE45 was determining the time steps after the first half-step was caculated.

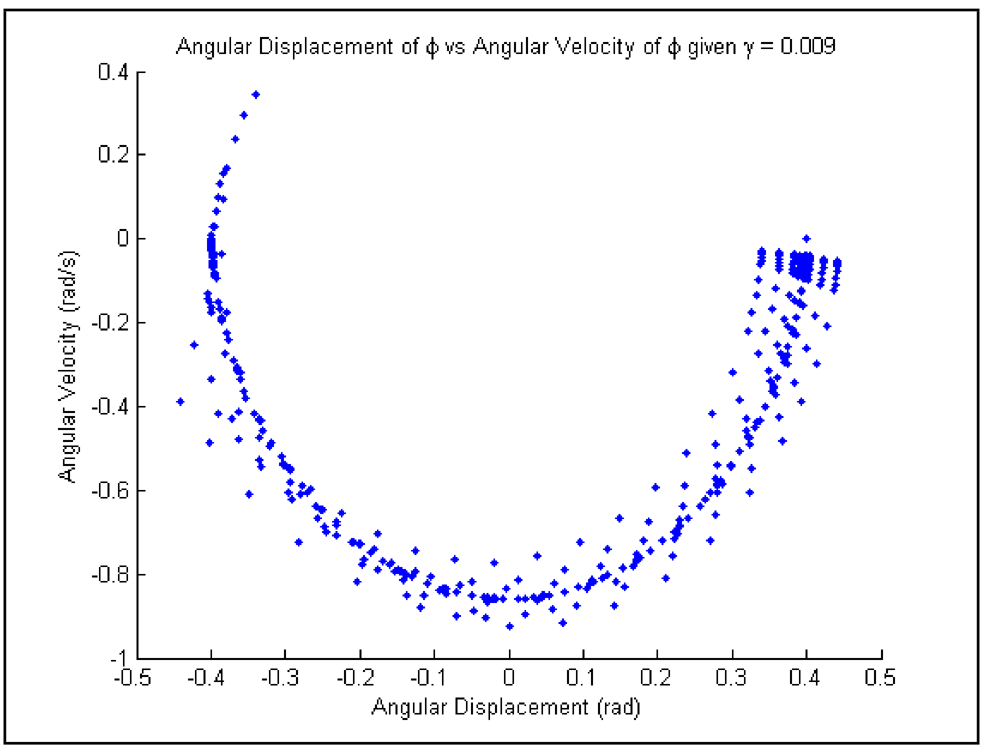


Figure 14: Plotting angular velocity of $\phi$ versus angular displacement of $\phi$ indicates that there were some inconsistencies in our results

### 4.4 Limitations of our Model

The use of our model is limited by its sensitivity to initial conditions and parameters. Although this is primarily due to the sensitivity of the physical system, our implemented model is slightly more sensitive than the model presented by Garcia et al. (This could possibly be remedied by refining the ODE step-size.) Additionally, our model cannot account for the contact between the walker legs and the ramp face, which limits its usefulness in modeling or planning the geometry of a physical walker. Finally, because it does not account for this contact, our model cannot accurately predict energy loss due to friction, making it of limited use when investigating efficiency.

## 5 Diagnosis

One of the initial roadblocks that we encountered during this project was the construction of the TinkerToy walker. We had originally wanted to use the TinkerToy walker to compare our simulated results with empirical ones, but were unable to get it stable and fully functional. The available TinkerToy parts have changed since Coleman and Ruina wrote their paper (i.e. the rods were different lengths, the blocks were different masses, etc), but we assumed that we could use these new parts interchangeably with those described in the paper. After failing to build a robust walker, we discovered in research that the success of these TinkerToy walkers is heavily dependant on having the same parts as described by Ruina and Coleman. When we discovered this, we decided to explore the peculiarities and limitations of the model. As it turns out, the sensitivity of the walkers to physical parameters is also demonstrated in our model quite effectively.

Determining parameters that are realistic and that work with the model was a significant challenge. The passive dynamic walker model that we used was far more sensitive to minute variations in initial conditions and physical parameters (eg: slope angle) than we had anticipated, so that we had difficulty choosing initial conditions and parameters that would allow our model to walk. This also made it difficult to ascertain whether our model was correct, as it could fail due to either poor code or poor parameters. We solved this problem by using initial conditions and parameters outlined in "The Simplest Walking Model: Stability, Complexity, and Scaling."

We also had an issue with triggering the event that stops the walking model. At first, we were stopping the ode45 calculating function whenever the following condition held beginequation $\phi(t) - 2\theta(t) = 0(1)$

We needed to stipulate that the ode45 calculating function stops only when this condition is true and the values of the calculation were decreasing. This kept us from triggering the event multiple times around the same heel strike due to the switch between swing and stance legs.

## 6 Improvement

This model could be improved by incorporating rigid body dynamics and accounting for the mass of the legs. Though our results were comparable to those found by Garcia et al., they would be necessarily less accurate when compared to experimental data from an actual passive dynamic walker due to the mass and moments of inertia of the legs. Additionally, this model could be improved by incorporating a knee-joint as described in . Because most passive dynamic walking robots have a knee joint, being able to simulate one would be very useful.

The final challenge we encountered in this project was creating the animation. We structured our code to be continuous (i.e. to calculate the angles $\theta$ and $\phi$) which unfortunately did not lend itself well to the animation of the situation. In the future, we would structure the code differently such that the animation of the situation for the hip (which we were able to complete) and the two feet could be completed.

## 7 Reflection

Throughout the course of this project, we learned a lot about the background and mechanics of passive dynamic walkers. Our research taught us a lot about the fundamental behaviors of unpowered bipedal motion, which we hadn't thought about in quite so much depth. As we had both spent this semester designing a powered bipedal robot for Mechanical Design, we found the behavior of passive dynamic walkers especially interesting. We had not expected the passive dynamic walker to be so dependent on its initial parameters, or that the behavior of the simplified model could be entirely characterized by ramp angle.

We were also mildly surprised that such a complex motion could be still-accurately described with such a simplified model but upon further reflection, it seemed somewhat on par for the course given some of the other simulations we have created in this class.

On the implementation side, we learned about the intricacies of events in MATLAB and how powerful they can be when dealing with ODE45.

# 8 Conclusion

Using the equations of motion provided by Garcia et al, we created a simulation of a passive dynamic walker. Within ramp angles of $0.007<\gamma<0.013$ radians, the system was stable and demonstrated realistic behavior that we validated by comparing to results from papers by McGreer and Garcia et al. With the current model, the passive dynamic walker is able to complete 16000 steps without falling. The MATLAB code is fast and fluid but has limitations in that it does not lend itself very well to animations and has in-determinant time steps. A few structural changes to the code would have improved the results of this project, however we still consider it a great learning experience.

# 9 Future Usage

We think that this project could be an appropriate project for future Dynamics classes. It addresses several concepts covered in the course, such as pendulum behavior, rotating reference frames, angular momentum balances, and event-driven ODE simulation and animation. It is also very scalable project, so that students with varying levels of comfort with Dynamics could still learn from it.

We would suggest to future groups however to work solely on the simulation of the system as passive dynamic walkers are very delicate systems. We would also suggest making mindful program structural choices. Unfortunately, we structured our code to be continuous as opposed to recording each step as something separate. Though this made the code faster and perhaps more intuitive, it did create a few issues when trying to animate the system.

A potential problem statement is: A rigid-leg passive dynamic walker (eg: TinkerToy model) makes its way down a small slope without falling. The slope length is at least ten times the stride length of the walker. Using "The Simplest Walking Model" paper for reference, develop a simulation for the walker and determine a possible range for the angle of this slope. Describe the behavior of state variables $\theta$, $\phi$, $\dot{\theta}$, and $\dot{\phi}$ over each step.

For bonus points, compare experimental data of a TinkerToy walker (gleaned from YouTube videos with a motion tracking program such as Tracker or LoggerPro) to your simulation.

# References


[1] Garcia, M., Chatterjee, A., Ruina, A., and Coleman, M., "The Simplest Walking Model: Stability, Complexity, and Scaling," *ASME Journal of Biomechanical Engineering*, Feb 1998.

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[3] Coleman, M, Ruina, A., "An Uncontrolled Toy That Can Walk But Cannot Stand Still," *Physical Review Letters*, pp.3658-3661, April 1998.



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